Q1

1

The point $P(-3, -2)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y - 2 = f(x) - 6$ $\quad y = f(x) - 4$

(ii) $y = f(x - 3)$

(iii) $2y = f(x)$ $\quad y = \frac{1}{2}f(x)$

(iv) $y = f\left(\frac{1}{2}x\right)$

[4]

i) VERTICAL TRANSLATION $\quad y$ CHANGES -4

$(-3, -2) \Rightarrow \boxed{(-3, -6)}$

ii) HORIZONTAL TRANSLATION $\quad x$ CHANGES $+3$

$(-3, -2) \Rightarrow \boxed{(0, -2)}$

iii) VERTICAL STRETCH $\quad y$ CHANGES $\times \frac{1}{2}$

$(-3, -2) \Rightarrow \boxed{(-3, -1)}$

iv) HORIZONTAL STRETCH $\quad x$ CHANGES $\times \frac{1}{a}$ $\times 2$

$(-3, -2) \Rightarrow \boxed{(-6, -2)}$

Q2

2

The point $P(0, 5)$ lies on the curve with equation $y = f(x)$.

State the coordinates of the image of point P on the curves with the following equations:

(i) $y = f(-x)$

(ii) $-y = f(x)$ $\quad y = -f(x)$

[2]

i) HORIZONTAL REFLECTION (y AXIS) $\quad x$ CHANGES

$(0, 5) \Rightarrow \boxed{(0, 5)}$

ii) VERTICAL REFLECTION (x AXIS) $\quad y$ CHANGES

$(0, 5) \Rightarrow \boxed{(0, -5)}$

Q3a

3a

The point $P(-12, -9)$ lies on the curve with equation $y = x^2 + 15x + 27$.

a) The graph is translated so that the point P is mapped to the point $(-12, 3)$. Write down the equation of the transformed function.

[2]

b) The graph is translated so that the point P is mapped to the point $(-10, -9)$. Write down the equation of the transformed function in the form $y = (x + a)^2 + 15(x + a) + 27$, where a is a constant to be found.

[2]

a) TRANSLATIONS $y = f(x + a)$ x COORDINATE CHANGES

VERTICAL $y = f(x) + a$ y COORDINATE CHANGES

$(-12, -9) \Rightarrow (-12, 3)$ $+12$ $\quad y = f(x) + 12$

$y = x^2 + 15x + 27 + 12$

$$\boxed{y = x^2 + 15x + 39}$$

Q3b

3b

The point $P(-12, -9)$ lies on the curve with equation $y = x^2 + 15x + 27$.

a) The graph is translated so that the point P is mapped to the point $(-12, 3)$. Write down the equation of the transformed function.

[2]

b) The graph is translated so that the point P is mapped to the point $(-10, -9)$. Write down the equation of the transformed function in the form $y = (x + a)^2 + 15(x + a) + 27$, where a is a constant to be found.

[2]

b) $y = f(x + a)$ HORIZONTAL TRANSLATION x CHANGES $-a$

$(-12, -9) \Rightarrow (-10, -9)$ $+2$

$a = -2$

$$\boxed{y = (x - 2)^2 + 15(x - 2) + 27}$$

Q4a

4a

The point $P(3, -12)$ lies on the curve with equation $y = x^2 - 12x + 15$.

a) The graph is stretched so that the point P is mapped to the point $(3, -4)$. Write down the equation of the transformed function in the form $y = ax^2 + bx + c$, where a, b and c are constants to be found.

[2]

b) The graph is stretched so that the point P is mapped to the point $(1, -12)$. Write down the equation of the transformed function in the form $y = (dx)^2 - 12(dx) + 15$, where d is a constant to be found.

[2]

a) VERTICAL STRETCH $y = af(x)$

y CHANGE $\times a$

HORIZONTAL STRETCH $y = f(ax)$

x CHANGE $\times \frac{1}{a}$

$(3, -12) \Rightarrow (3, -4)$

$\times \frac{1}{3}$

$a = \frac{1}{3}$

$y = \frac{1}{3}(x^2 - 12x + 15)$

$$\boxed{y = \frac{1}{3}x^2 - 4x + 5}$$

Q4b

4b

The point $P(3, -12)$ lies on the curve with equation $y = x^2 - 12x + 15$.

a) The graph is stretched so that the point P is mapped to the point $(3, -4)$. Write down the equation of the transformed function in the form $y = ax^2 + bx + c$, where a, b and c are constants to be found.

[2]

b) The graph is stretched so that the point P is mapped to the point $(1, -12)$. Write down the equation of the transformed function in the form $y = (dx)^2 - 12(dx) + 15$, where d is a constant to be found.

[2]

b) STRETCH $y = af(x)$ OR $y = f(ax)$

HORIZONTAL STRETCH

x CHANGES $\times \frac{1}{a}$

OR $\div a$

$(3, -12) \Rightarrow (1, -12)$

$\div 3$ OR $\times \frac{1}{3}$

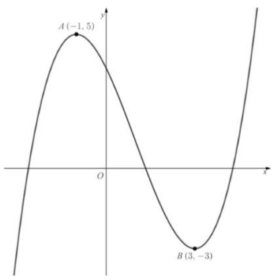
$d = 3$

$$\boxed{y = (3x)^2 - 12(3x) + 15}$$

Q5a

5a

The diagram below shows the graph of $y = f(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.



(a) In separate diagrams, sketch the curves with equation

(i) $y = f(-x)$ HORIZONTAL REFLECTION (Y AXIS) x CHANGE

(ii) $-y = f(x)$ $y = -f(x)$ VERTICAL REFLECTION (x AXIS) y CHANGE

On each diagram, give the coordinates of the images of points A and B under the given transformation.

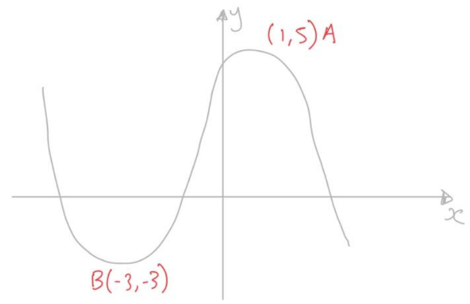
[4]

(b) On the graph of $y = f(x + a)$ the images of the two marked points both lie on the same side of the y-axis. Find the range of possible values of a.

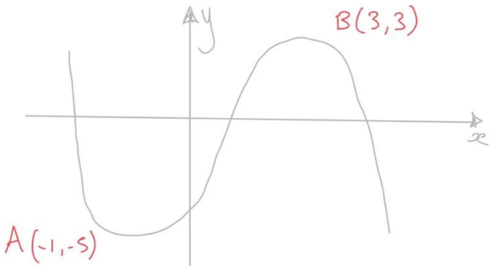
[3]

a)

i)



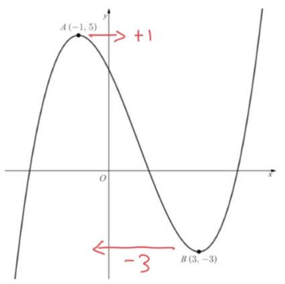
ii)



Q5b

5b

The diagram below shows the graph of $y = f(x)$. The two marked points $A(-1, 5)$ and $B(3, -3)$ lie on the graph.



(a) In separate diagrams, sketch the curves with equation

(i) $y = f(-x)$

(ii) $-y = f(x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation.

[4]

(b) On the graph of $y = f(x + a)$ the images of the two marked points both lie on the same side of the y-axis. Find the range of possible values of a.

[3]

b) HORIZONTAL TRANSLATION x CHANGE $-a$

$A(-1, 5) \Rightarrow$ RIGHT BY MORE THAN 1

$a < -1$

$B(3, -3) \Rightarrow$ LEFT BY MORE THAN 3

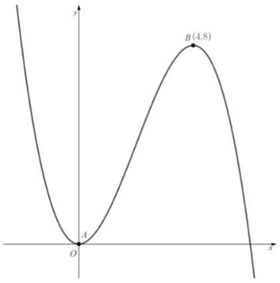
$a > 3$

$\boxed{a < -1 \quad \text{OR} \quad a > 3}$

Q6

6

The diagram below shows the graph of $y = \mathrm{f}(x)$. The marked point $B(4,8)$ lies on the graph, and the graph meets the origin at the marked point A.



In separate diagrams, sketch the curves with equation

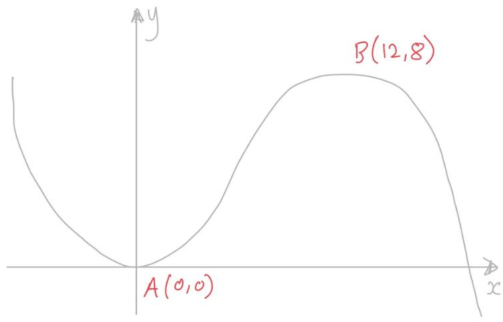
(i) $y = \mathrm{f}\left(\frac{1}{3}x\right)$ HORIZONTAL STRETCH x CHANGE $\times \frac{1}{a}$ $\times 3$

(ii) $6y = \mathrm{f}(x)$ $y = \frac{1}{6}\mathrm{f}(x)$ VERTICAL STRETCH y CHANGE $\times \frac{1}{6}$

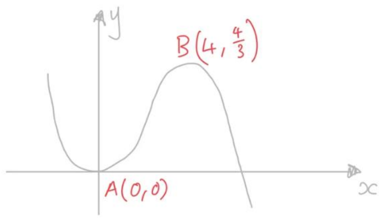
On each diagram, give the coordinates of the images of points A and B under the given transformation.

[4]

i)



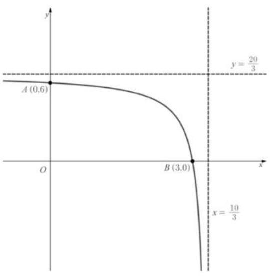
ii)



Q7a

7a

The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph intersects the coordinate axes at the two marked points $A(0,6)$ and $B(3,0)$. The graph has two asymptotes as shown, with equations $y = \frac{20}{3}$ and $x = \frac{10}{3}$.



(a) In separate diagrams, sketch the curves with equation

(i) $y = \mathrm{f}(5x)$ HORIZONTAL STRETCH x CHANGE $\times \frac{1}{5}$ OR $\div 5$

(ii) $y = -\mathrm{f}(x)$ REFLECTION VERTICAL (x AXIS) y CHANGE

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

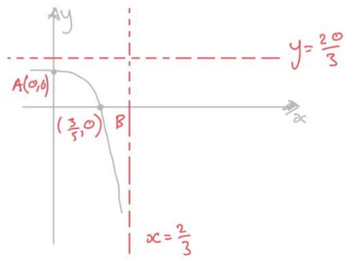
[6]

(b) The graph of $y = a\mathrm{f}(x)$ has an asymptote with equation $y = 2$. Find the value of a.

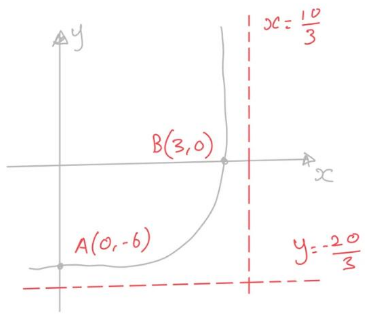
[1]

a)

i)



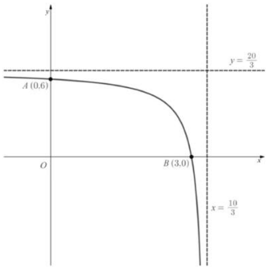
ii)



Q7b

7b

The diagram below shows the graph of $y = \mathrm{f}(x)$. The graph intersects the coordinate axes at the two marked points $A(0, 6)$ and $B(3, 0)$. The graph has two asymptotes as shown, with equations $y = \frac{20}{3}$ and $x = \frac{10}{3}$.



(a) In separate diagrams, sketch the curves with equation

(i) $y = \mathrm{f}(5x)$

(ii) $y = -\mathrm{f}(x)$

On each diagram, give the coordinates of the images of points A and B under the given transformation, as well as stating the equations of the transformed asymptotes.

[6]

(b) The graph of $y = a\mathrm{f}(x)$ has an asymptote with equation $y = 2$. Find the value of a.

[1]

b) VERTICAL STRETCH $\quad$ y CHANGES $\times a$

$y = \frac{20}{3} \Rightarrow y = 2$

$\times a$

$a = 2 \div \frac{20}{3}$

$\boxed{a = \frac{3}{10}}$

Q8a

8a

(a) Sketch the graph of $y = 2 - \frac{8}{x^2}$, showing clearly the points where the curve crosses the coordinate axes and stating the equations of the asymptotes.

[4]

(b) The graph of $y = 2 - \frac{8}{(x+a)^2}$ passes through the origin. Find the two possible values of a.

[2]

$\frac{a}{x}$

RECIPROCAL

$\frac{a}{x^2}$

RECIPROCAL²

$-\frac{a}{x^2}$

NEGATIVE RECIPROCAL²

+2 VERTICAL TRANSLATION $\quad$ y CHANGES

x INTERCEPTS $\quad y = 0$

$0 = 2 - \frac{8}{x^2}$

$\frac{8}{x^2} = 2$

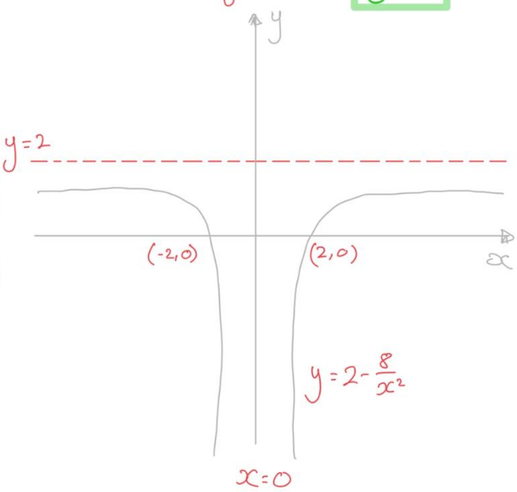
$x^2 = \frac{8}{2} = 4$

$x = \pm\sqrt{4} = \pm 2$

$\boxed{(-2,0)}$
$\boxed{(2,0)}$

a) ASYMPTOTES $\quad x = 0 \Rightarrow \boxed{x = 0}$

$y = 0 \Rightarrow \boxed{y = 2}$



Q8b

8b

(a) Sketch the graph of $y = 2 - \frac{8}{x^2}$, showing clearly the points where the curve crosses the coordinate axes and stating the equations of the asymptotes. [4]

(b) The graph of $y = 2 - \frac{8}{(x+a)^2}$ passes through the origin. Find the two possible values of a. [2]

b) ORIGIN = $(0,0)$

SOLVE WHERE $x=0$ AND $y=0$

$$0 = 2 - \frac{8}{(0+a)^2}$$

$+\frac{8}{a^2}$

$$\frac{8}{a^2} = 2$$

$\times a^2$

$$8 = 2a^2$$

$\div 2$

$$4 = a^2$$

$\sqrt{\ }$

$$a = \pm\sqrt{4} = \pm 2$$

$$\boxed{a = 2 \quad \text{OR} \quad a = -2}$$

Q9a

9a

Given that $x^3 - 8x^2 + 16x = x(x-4)^2$

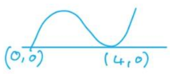
(a) Sketch the graph of $y = x^3 - 8x^2 + 16x + 3$, showing clearly the coordinates of the points where the curve crosses the coordinate axes and the co-ordinates of any minimum points. (You do not need to state the co-ordinates of any maximum points).

[4]

(b) The graph with equation $y + a = x^3 - 8x^2 + 16x$ crosses the x-axis three times. Find the range of possible values of a. [2]

FOR $x^3 - 8x^2 + 16x = x(x-4)^2$

$x = 0$ $\quad x = 4$ (REPEATED ROOT)



$y = f(x) + 3$ VERTICAL TRANSLATION y CHANGES $+3$

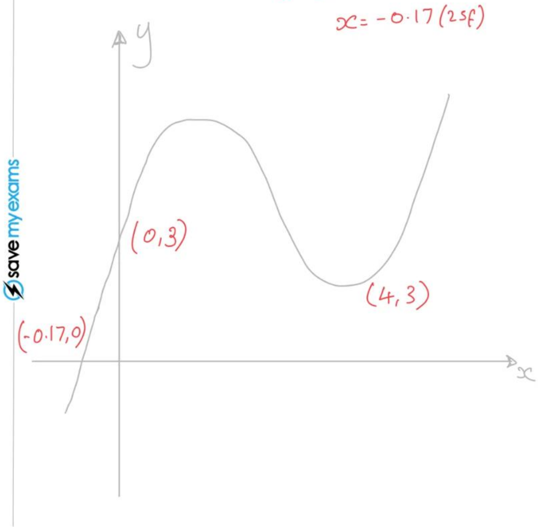
$(0,0) \Rightarrow \boxed{(0,3)}$

$(4,0) \Rightarrow \boxed{(4,3)}$

a) x INTERCEPT $y = 0$

$x^3 - 8x^2 - 16x + 3 = 0$

$x = -0.17$ (2sf)



Q9b

9b

Given that $x^3 - 8x^2 + 16x = x(x-4)^2$

(a) Sketch the graph of $y = x^3 - 8x^2 + 16x + 3$, showing clearly the coordinates of the points where the curve crosses the coordinate axes and the co-ordinates of any minimum points. (You do not need to state the co-ordinates of any maximum points).

[4]

(b) The graph with equation $y + a = x^3 - 8x^2 + 16x$ crosses the x-axis three times. Find the range of possible values of a.

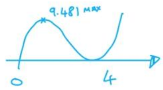
[2]

b) $y + a = f(x)$

$y = f(x) - a$ VERTICAL TRANSLATION $-a$

$f(x) = x(x-4)^2$

$x = 0 \quad x = 4$



$y = f(x) - a$

ANY VERTICAL TRANSLATION DOWN BY a WILL HAVE THREE SOLUTIONS UNTIL MAX IS REACHED AT 9.481 (USE CALC TO SOLVE)

$\boxed{0 < a < 9.481}$